Incompatibility of simultaneous nonlinear realizations of scale symmetry and supersymmetry

T. E. Clark* and S. T. Love†
Department of Physics, Purdue University, West Lafayette, Indiana 47907-1396
(Received 29 June 1999; published 7 February 2000)

Simultaneous nonlinear realizations of supersymmetry and softly broken scale and chiral symmetries are investigated. To guarantee Nambu-Goldstone realizations of the symmetries, the Goldstino decay constant is forced to vary as the explicit soft scale and chiral symmetry breaking parameters. Consequently, it must vanish in the chiral limit and the simultaneous nonlinear realizations of the super and scale symmetries prove inconsistent.

PACS number(s): 11.30.Pb, 11.30.Ly, 11.30.Qc, 14.80.Mz

Goldstone's theorem [1] guarantees that associated with every spontaneously broken global symmetry there is a massless particle. Below the symmetry breaking scale, the dynamics of these Nambu-Goldstone degrees of freedom can be described by an action which realizes the spontaneously broken symmetry nonlinearly. This effective action encapsulates all the consequences of the symmetry current algebra and, through lowest non-trivial order in a derivative expansion, is unique up to reparametrization, independent of the underlying theory [2,3].

For the case of an internal global symmetry group G spontaneously broken to an invariant subgroup H, the Nambu-Goldstone fields, π^i, $i=1,\ldots \dim G/H$, act as coordinates of the coset manifold G/H. A particular choice of the coset coordinates is the standard realization parametrized as

$$U(\pi)=e^{2iT^i\pi^i/F_\pi}, \tag{1}$$

where T^A is the fundamental representation of G and F_π is the Nambu-Goldstone boson decay constant. While U transforms linearly under G, the Nambu-Goldstone fields, π^i, transform linearly only under H and nonlinearly under the spontaneously broken G generators. With this choice of coordinates, the G invariant effective Lagrangian [4] is simply

$$\mathcal{L}=\frac{F_\pi^2}{4}\mathrm{Tr}[\partial_\mu U^\dagger \partial^\mu U]. \tag{2}$$

In addition to the spontaneous symmetry breaking, there also often appears some soft explicit symmetry breaking. For the case of chiral symmetry breaking, this explicit breaking takes the form of a soft mass term and the above effective Lagrangian is modified to

$$\mathcal{L}=\frac{F_\pi^2}{4}\mathrm{Tr}[\partial_\mu U^\dagger \partial^\mu U]-uF_\pi^2\mathrm{Tr}[mU^\dagger+Um]. \tag{3}$$

Here m is the mass matrix characterizing the soft explicit breaking and u is the order parameter of the spontaneous symmetry breaking. For example, if the chiral symmetry is dynamically broken due to some underlying strong gauge interaction, then $u=\langle\bar{\psi}\psi\rangle/2F_\pi^2$, where ψ is a chiral fermion of the underlying theory. The above effective Lagrangian assumes that the theory is free of chiral anomalies. If such effects are also present, the effective action is further modified by the inclusion of a Wess-Zumino term [5–7]. While we have focused on the case of a single spontaneously broken global symmetry, the analysis is trivially extended to include the possibility of multiple spontaneously and softly broken internal global symmetry groups. The resultant effective Lagrangian is simply obtained by additively including the individual effective Lagrangians.

In addition to the case of spontaneously broken global symmetries, one can also construct effective Lagrangians which nonlinearly realize scale symmetry. Thus we envision an underlying model where over some energy range the quantum fluctuations are such that the scale anomaly either vanishes or is but a very small effect and dilatations are approximately a good symmetry broken only by some soft mass terms. Then when a global internal symmetry is spontaneously broken, there will be an accompanying spontaneous scale symmetry breakdown [8–10] and the spectrum will include its associated Nambu-Goldstone boson, the dilaton D. Since, in general, the couplings of the underlying theory do indeed run, the dilaton is a pseudo Nambu-Goldstone boson acquiring a mass related to the scale at which the renormalization group β functions become significant. To nonlinearly realize the scale symmetry, one introduces a standard realization for the dilaton as

$$S(D)=e^{D/F_D} \tag{4}$$

where F_D is the dilaton decay constant. The associated scale transformation, parametrized by ϵ, is

$$\delta^D(\epsilon)S=\epsilon(1+x^\nu\partial_\nu)S \tag{5}$$

and results in an inhomogeneous scale transformation of the dilaton as

$$\delta^D(\epsilon)D=\epsilon(F_D+x^\nu\partial_\nu D). \tag{6}$$

Since the generators of space-time scale transformations commute with those of the internal symmetry group G, the Nambu-Goldstone boson fields π^i are constrained [11] to carry zero scale weight

*Email address: clark@physics.purdue.edu
†Email address: love@physics.purdue.edu

$$\delta^D(\epsilon)\pi^i=\epsilon x^\nu\partial_\nu\pi^i, \tag{7}$$

while the dilaton is a G singlet and thus satisfies

$$\delta^G(\omega)D=0, \tag{8}$$

with ω^A parametrizing the group G transformations.

The effective action (3) can be made scale invariant up to soft breaking terms by including appropriate powers of S to make the scaling weight of each invariant term be four. The soft explicit scale and G symmetry breaking terms are dictated by the form of the underlying theory. For the case of a global chiral symmetry broken by a soft fermion mass term which also softly breaks the scale symmetry, the effective Lagrangian which nonlinearly realizes both the chiral and the scale symmetry takes the form [12,13]

$$\begin{aligned}\mathcal{L}=&\frac{1}{2}F_D^2\partial_\mu S\partial^\mu S+\frac{1}{4}F_\pi^2S^2\mathrm{Tr}[\partial_\mu U^\dagger\partial^\mu U]\\&-\frac{1}{2}uF_\pi^2(3-\gamma)S^4\mathrm{Tr}[m]\\&+uF_\pi^2S^{3-\gamma}\mathrm{Tr}[mU^\dagger+Um],\end{aligned} \tag{9}$$

where γ is the anomalous dimension of the underlying fermion field. Under the nonlinear scale and chiral symmetries, this effective Lagrangian transforms as

$$\delta^D(\epsilon)\mathcal{L}=\epsilon(3+x^\nu\partial_\nu)\{uF_\pi^2(3-\gamma)S^{3-\gamma}\mathrm{Tr}[mU^\dagger+Um]\} \tag{10}$$

$$\delta^G(\omega)\mathcal{L}=-i\omega^AuF_\pi^2S^{3-\gamma}\mathrm{Tr}[T^A(mU^\dagger-Um)], \tag{11}$$

where the right hand sides reflect the soft explicit symmetry breaking.

The fact that the coefficient of the scale and chirally invariant S^4 term in the Lagrangian (9) depends on the explicit breaking mass parameter, m, is somewhat unusual and warrants some further elaboration. The necessity for this value can be established by expanding $\mathcal{L}$ in powers of the dilaton field D. The elimination of the destabilizing term linear in D is accomplished by fixing it to be $-\frac{1}{2}uF_\pi^2(3-\gamma)\mathrm{Tr}[m]$ as in Eq. (9). The dependence of the S^4 coefficient on the explicit scale and chiral symmetry breaking parameter m is dictated in order for the symmetry to be realized in the manner of Nambu-Goldstone with $\langle 0|S|0\rangle=1$ and $\langle 0|U|0\rangle=I$ so that $\langle 0|D|0\rangle=0$ and $\langle 0|\pi^i|0\rangle=0$. The vanishing of the S^4 coupling in the chiral limit, $m\to 0$, is required since a potential of the form λS^4 with λ nonvanishing gives a classical vacuum corresponding to $\langle S\rangle_0=0$ which drives $\langle D\rangle_0\to-\infty$. This instability signals that the corresponding effective Lagrangian realizes the symmetry in a Wigner-Weyl mode. Consequently, a Nambu-Goldstone realization of the symmetry requires a vanishing of the S^4 coupling in the chiral limit, $m\to 0$. On the other hand, one cannot simply ignore the S^4 term entirely since in its absence, the dilaton becomes tachyonic for $m\neq 0$. In the exact symmetry chiral limit, $m\to 0$, the invariant effective Lagrangian is simply obtained as

$$\mathcal{L}=\frac{1}{2}F_D^2\partial_\mu S\partial^\mu S+\frac{1}{4}F_\pi^2S^2\mathrm{Tr}(\partial_\mu U^+\partial^\mu U). \tag{12}$$

Let us next consider nonlinear realizations of another type of space-time symmetry, namely supersymmetry (SUSY). For spontaneously broken SUSY, the dynamics of the Nambu-Goldstone fermion, the Goldstino, is described by the Akulov-Volkov Lagrangian [14]. The nonlinear SUSY transformations of the Goldstino fields are given by

$$\begin{aligned}\delta^Q(\xi,\bar{\xi})\lambda^\alpha&=F\xi^\alpha+\Lambda^\rho(\xi,\bar{\xi})\partial_\rho\lambda^\alpha\\\delta^Q(\xi,\bar{\xi})\bar{\lambda}_{\dot\alpha}&=F\bar{\xi}_{\dot\alpha}+\Lambda^\rho(\xi,\bar{\xi})\partial_\rho\bar{\lambda}_{\dot\alpha},\end{aligned} \tag{13}$$

where ξ^α, $\bar{\xi}_{\dot\alpha}$ are the Weyl spinor SUSY transformation parameters and $\Lambda^\rho(\xi,\bar{\xi})\equiv-(i/F)(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})$ is a Goldstino field dependent translation vector and F is the Goldstino decay constant. The Akulov-Volkov Lagrangian takes the form

$$\mathcal{L}_{AV}=-\frac{F^2}{2}\det A, \tag{14}$$

with $A_\mu^\nu=\delta_\mu^\nu+(i/F^2)\lambda\overleftrightarrow{\partial}_\mu\sigma^\nu\bar{\lambda}$ the Akulov-Volkov vierbein. Under the nonlinear SUSY variations, it transforms as the total divergence

$$\delta^Q(\xi,\bar{\xi})\mathcal{L}_{AV}=\partial_\rho(\Lambda^\rho\mathcal{L}_{AV}), \tag{15}$$

and hence the associated action $I_{AV}=\int d^4x\mathcal{L}_{AV}$ is SUSY invariant.

Finally, let us explore the possibility of having simultaneous nonlinear realizations of scale, chiral, and super symmetries. The superconformal algebra [15,16] nontrivially relates the SUSY and scale symmetry transformations as

$$[\delta^D(\epsilon),\delta^Q(\xi,\bar{\xi})]=\frac{1}{2}\delta^Q(\epsilon\xi,\epsilon\bar{\xi}). \tag{16}$$

As a consequence, the Goldstino fields transform with scaling weight $-\frac{1}{2}$ (recall the Nambu-Goldstone bosons of the spontaneously broken global symmetry carry scaling weight 0) as

$$\begin{aligned}\delta^D(\epsilon)\lambda_\alpha&=\epsilon\left(-\frac{1}{2}+x^\nu\partial_\nu\right)\lambda_\alpha\\\delta^D(\epsilon)\bar{\lambda}^{\dot\alpha}&=\epsilon\left(-\frac{1}{2}+x^\nu\partial_\nu\right)\bar{\lambda}^{\dot\alpha}.\end{aligned} \tag{17}$$

As is the case for any matter field, the SUSY is nonlinearly realized [17,19] on the dilaton field as

$$\delta^Q(\xi,\bar{\xi})D=\Lambda^\rho(\xi,\bar{\xi})\partial_\rho D. \tag{18}$$

We now attempt to construct an effective Lagrangian containing the Goldstino, dilaton, and the Nambu-Goldstone bosons of the global symmetry group G which non-linearly realizes the scale, G, and super symmetries, up to some soft explicit breakings of the scale and G symmetries. To render the Akulov-Volkov Lagrangian scale invariant, we simply multiply it by S^4 to raise its scaling weight to four. On the other hand, the scale and G invariant effective Lagrangian pieces can be made nonlinearly SUSY invariant [18,19] by simply multiplying them by the Akulov-Volkov determinant, $\det A$, and replacing all space-time derivatives by nonlinear SUSY covariant derivatives so that, for example, $\partial_\mu \pi^i \to \mathcal{D}_\mu \pi^i = (A^{-1})_\mu{}^\nu \partial_\nu \pi^i$. So doing, we secure the effective Lagrangian

$$\mathcal{L} = -\frac{F^2}{2}(\det A)S^4 + \frac{F_D}{2}(\det A)\mathcal{D}_\mu S \mathcal{D}^\mu S + \frac{F_\pi^2}{4}(\det A)S^2 \mathrm{Tr}[\mathcal{D}_\mu U^\dagger \mathcal{D}^\mu U] + uF_\pi^2(\det A)S^{3-\gamma}\mathrm{Tr}[mU^\dagger + Um], \tag{19}$$

where the last term is a soft explicit chiral and scale symmetry breaking term.

However, noting that $\det A$ starts with unity, then in order to guarantee that the scale symmetry be realized in a Nambu-Goldstone manner, the coefficient of the S^4 term which is the Goldstino decay constant cannot be chosen arbitrarily, but instead must be proportional to the soft explicit chiral and scale symmetry breaking parameters. Explicitly the Nambu-Goldstone alternative requires $\langle 0|D|0\rangle = 0$. Hence the coefficient of the term linear in the dilaton field in Eq. (19) must vanish, requiring

$$F^2 = (3-\gamma)uF_\pi^2 \mathrm{Tr}[m]. \tag{20}$$

The necessity of this identification is identical to that in the non-SUSY case. Unless this coefficient vanishes in the good symmetry chiral limit, the dilaton vacuum value is driven to negative infinity.

Of course, the fact that the Goldstino decay constant (and hence the SUSY breaking scale) vanishes in the chiral limit raises its own host of complications. First and foremost, it implies that the scale symmetry and SUSY cannot both be simultaneously realized as nonlinear symmetries. Note that there is no difficulty in constructing an effective Lagrangian invariant under both nonlinear SUSY and chiral symmetry transformations. It is only when both SUSY and scale symmetry are to be realized nonlinearly that one encounters the inconsistency. The source of the problem can be directly traced to the fact that the Akulov-Volkov Lagrangian relates the coefficients of the derivatively coupled Goldstino self interactions and an overall constant which is the vacuum energy accompanying the spontaneous superymmetry breaking. This constant vacuum energy term is ignorable until one makes the model nonlinearly scale invariant, which is achieved by multiplying the entire Akulov-Volkov Lagrangian by S^4. So doing, not only do the Goldstino kinetic term and its self interactions get multiplied by this factor, but so does the constant term. As such the erstwhile constant term now becomes a dilaton self interaction potential term. In order for the dilaton to be a Nambu-Goldstone particle, it cannot sustain a potential whose coefficients are nonvanishing in the chiral limit. Thus the coefficient of this term which is the Goldstino decay constant must vanish in the chiral limit. But this is the same coefficient as that multiplying the entire Akulov-Volkov determinant. Since the Goldstino decay constant is the matrix element of the supersymmetry current between the vacuum and the Goldstino state, its vanishing signals the nonviability of the Goldstone realization. Consequently, both symmetries cannot be simulataneously realized nonlinearly and the spectrum cannot contain both a dilaton and a Goldstino as Nambu-Goldstone particles. We have been unable to find a way out of this conundrum. One may try to include additional low energy degrees of freedom and see if the resulting model is consistent. Since R symmetry is another component of the superconformal algebra, we have investigated also including an R-axion, the Nambu-Goldstone boson of spontaneous broken R symmetry. Once again the resultant effective Lagrangian does not have a consistent interpretation in the chiral limit. It thus appears as if spontaneous supersymmetry breaking requires the presence of explicit (hard) scale symmetry breaking. This is certainly the case for the various models which have been studied in the literature [20,21]. Here we have provided an argument that this is actually the case in general. Note that the literature contains numerous studies of models purporting to include both dilatons and Goldstinos. In all of these models, however, the dilaton acquires a non-zero vacuum expectation value. Consequently, it is not really a Nambu-Goldstone boson and there is no contradiction with our result. Finally, we note that when the supersymmetry is made local and the super Higgs mechanism is implemented, the Goldstino is absorbed into the gravitino rendering it massive. A resulting supergravity model can be constructed [22] such that the cosmological constant is cancelled by a term arising from the supergravity sector.

This work was supported in part by the U.S. Department of Energy under grant DE-FG02-91ER40681 (Task B).

[1] J. Goldstone, Nuovo Cimento **19**, 154 (1961).
[2] S. Weinberg, Physica A **96**, 327 (1979).
[3] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2239 (1969); C. Callen, S. Coleman, J. Wess, and B. Zumino, *ibid.* **177**, 2247 (1969).
[4] See, for example, H. Georgi, *Weak Interactions and Modern Particle Theory* (Benjamin/Cummings, New York, 1984).
[5] J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971).
[6] E. Witten, Nucl. Phys. **B223**, 422 (1984).
[7] T.E. Clark and S.T. Love, Phys. Lett. **138B**, 289 (1984).
[8] W.A. Bardeen, C.N. Leung, and S.T. Love, Phys. Rev. Lett. **56**, 1230 (1986).
[9] W.A. Bardeen, C.N. Leung, and S.T. Love, Nucl. Phys. **B273**, 649 (1986).
[10] W.A. Bardeen and S.T. Love, Phys. Rev. D **45**, 4672 (1992).

[11] T.E. Clark, C.N. Leung, and S.T. Love, Phys. Rev. D **35**, 997 (1987); T. E. Clark and S. T. Love, Int. J. Mod. Phys. A **11**, 2807 (1996).

[12] W.A. Bardeen, in *New Trends in Strong Coupling Gauge Theories*, Proceedings of the 1988 International Workshop, Nagoya, Japan, edited by M. Bando *et al.* (World Scientific, Singapore, 1989), p. 110.

[13] W.A. Bardeen, C.N. Leung, and S.T. Love, Nucl. Phys. **B323**, 493 (1989).

[14] V.P. Akulov and D.V. Volkov, Pis'ma Zh. Éksp. Teor. Fiz. **16**, 621 (1972) [JETP Lett. **16**, 438 (1972)].

[15] J. Wess and B. Zumino, Nucl. Phys. **B70**, 39 (1974).

[16] S. Ferrara, Nucl. Phys. **B77**, 73 (1974).

[17] T.E. Clark and S.T. Love, Phys. Rev. D **39**, 2391 (1989).

[18] T.E. Clark and S.T. Love, Phys. Rev. D **54**, 5723 (1996).

[19] T.E. Clark, T. Lee, S.T. Love, and G.-H. Wu, Phys. Rev. D **57**, 5912 (1998).

[20] I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. **B256**, 557 (1985).

[21] A. Nelson and N. Seiberg, Nucl. Phys. **B416**, 46 (1994).

[22] S. Deser and B. Zumino, Phys. Rev. Lett. **38**, 1433 (1977); E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello, and P. van Nieuwenhuizen, Phys. Lett. **79B**, 231 (1978); Nucl. Phys. **B147**, 105 (1979).